Write a linear system for this problem and solve it using Gauss-Jordan elimination (pivot method). **SCORE: _____ / 10 PTS**

Lee is buying supplies for a design project. Beige tiles sell for $3 each, and shipping costs are $1 per tile. Stone tiles sell for $5 each, and shipping costs are $2 per tile. Simulated leather tiles sell for $8 each, and shipping costs are $3 per tile. Each tile weighs 10 ounces (beige), 12 ounces (stone) and 9 ounces (simulated leather). Lee places an order weighing 269 ounces, which costs $48 for shipping and $129 for the tiles. How many of each type of tile does Lee buy ?

NOTE: You must state the row operations you used (as demonstrated in class).
As part of your work, you must produce a matrix in *reduced* row echelon form.
You must check your final answer.

$$10b+12s+9l=269$$
$$b+2s+3l=48$$
$$3b+5s+8l=129$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 48 \\ 3 & 5 & 8 & 129 \\ 10 & 12 & 9 & 269 \end{array}\right] \begin{array}{l} \\ R_2+(-3)R_1 \\ R_3+(-10)R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 48 \\ 0 & -1 & -1 & -15 \\ 0 & -8 & -21 & -211 \end{array}\right] -R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 48 \\ 0 & 1 & 1 & 15 \\ 0 & -8 & -21 & -211 \end{array}\right] R_3+8R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 48 \\ 0 & 1 & 1 & 15 \\ 0 & 0 & -13 & -91 \end{array}\right] -\tfrac{1}{13}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 48 \\ 0 & 1 & 1 & 15 \\ 0 & 0 & 1 & 7 \end{array}\right] \begin{array}{l} R_1+(-3)R_3 \\ R_2+(-1)R_3 \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 27 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7 \end{array}\right] R_1+(-2)R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

CHECK:
$$110+96+63=269$$
$$11+16+21=48$$
$$33+40+56=129$$

11 BEIGE, 8 STONE,
7 SIMULATED LEATHER

Let $R=\begin{bmatrix} -2 & 0 & 2 & -1 \\ 3 & -1 & 1 & -2 \\ 1 & 2 & 0 & 4 \end{bmatrix}$ and $S=\begin{bmatrix} 0 & -1 \\ 1 & 2 \\ -2 & 1 \\ 3 & 0 \end{bmatrix}$. Which product, RS or SR, exists ? Find its value. **SCORE: _____ / 4 PTS**

3×4 $\quad$ 4×2

$$\underset{3\times 2}{RS}=\begin{bmatrix} -4-3 & 2+2 \\ -1-2-6 & -3-2+1 \\ 2+12 & -1+4 \end{bmatrix}=\begin{bmatrix} -7 & 4 \\ -9 & -4 \\ 14 & 3 \end{bmatrix}$$

FILL IN THE BLANKS. **SCORE: _____ / 3 PTS**

If matrix A has 7 columns, matrix B has 6 rows, matrix C has 9 rows, and $A = CB$, then the order of matrix A is $\underline{9 \times 7}$, the order of matrix B is $\underline{6 \times 7}$, the order of matrix C is $\underline{9 \times 6}$.

Solve the linear system
$$\begin{aligned} 3x + 5y - 9z &= -13 \\ -4x - 5y + 7z &= 9 \\ -x - 2y + 4z &= 6 \end{aligned}$$
using Gauss-Jordan elimination (pivot method). **SCORE: _____ / 7 PTS**

NOTE: You must state the row operations you used (as demonstrated in class).
As part of your work, you must produce a matrix in *reduced* row echelon form.
You do NOT need to check your final answer.

$$\left[\begin{array}{ccc|c} 3 & 5 & -9 & -13 \\ -4 & -5 & 7 & 9 \\ -1 & -2 & 4 & 6 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} -1 & -2 & 4 & 6 \\ -4 & -5 & 7 & 9 \\ 3 & 5 & -9 & -13 \end{array}\right] -R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -6 \\ -4 & -5 & 7 & 9 \\ 3 & 5 & -9 & -13 \end{array}\right] \begin{array}{l} R_2 + 4R_1 \\ R_3 + (-3)R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -6 \\ 0 & 3 & -9 & -15 \\ 0 & -1 & 3 & 5 \end{array}\right] \tfrac{1}{3}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -6 \\ 0 & 1 & -3 & -5 \\ 0 & -1 & 3 & 5 \end{array}\right] R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -6 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right] R_1 + (-2)R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x = -2z + 4$
$y = 3z - 5$
$z = z$

Find $\begin{vmatrix} -2 & 2 & 9 & 4 & 0 \\ 3 & 2 & 0 & -3 & 1 \\ -3 & -1 & -6 & 2 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ -2 & 7 & 5 & 1 & 3 \end{vmatrix}$. (HINT: The answer is between -50 and 50.) **SCORE: _____ / 6 PTS**

$$= -2 \begin{vmatrix} -2 & 2 & 4 & 0 \\ 3 & 2 & -3 & 1 \\ -3 & -1 & 2 & 0 \\ -2 & 7 & 1 & 3 \end{vmatrix} = -2 \left(1 \begin{vmatrix} -2 & 2 & 4 \\ -3 & -1 & 2 \\ -2 & 7 & 1 \end{vmatrix} + 3 \begin{vmatrix} -2 & 2 & 4 \\ 3 & 2 & -3 \\ -3 & -1 & 2 \end{vmatrix} \right)$$

$$= -2 \left(\begin{array}{l} -2(-15) - 2(1) + 4(-23) \\ + 3(-2(1) - 2(-3) + 4(3)) \end{array} \right)$$

$$= -2(30 - 2 - 92 - 6 + 18 + 36)$$

$$= -2(-64 + 48) = -2(-16) = 32$$